

COMPARATIVE STUDY REGARDING THE OBTAINMENT OF COORDINATES FOR THE SUPPORT POINTS USING THE TAVERSE METHOD AND THE METHOD OF THE LEAST SQUARES

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ABSTRACT

A well known truth states that the position (of coordinates) for the new points is determined through several methods: intersections, double radiations, traversing, or the method of the least squares. The new points are being used by specialists in order to create topographical upheavals or cadastral plans. Lately, the great majority of specialists create works for the property right tabulation over small surfaces. Considering the development of the GNSS technology, two GNSS receptor points are determined for the support points from which the traversing is being drawn. Theoretically traversing is not the best method. It is recommended to use the method of the least squares since this takes into account all the measurements conducted. This statement will be demonstrated in this study.

GENERAL CONSIDERATIONS

Lately, the topographic instruments have evolved. Ever since the emergence of the total stations and the GPS technology, subsequently turned into GNSS, performances have been registered that were difficult to reach otherwise.

Thus, we can now talk about millimeter precisions, and millimeter errors. If we look at the data shown by the reports after the processing of the GNSS data, we can say that the position is given with a precision centimeter level, for the least. From the perspective of the used devices these precisions are credible, considering their performances. From the perspective of data processing however, it is questionable whether the precisions are real, and whether the coordinates are the closest possible to the real value.

This is not always the case, we start from the GNSS receptors that produce very good positions in the system WGS84, but which need to be transformed in a plane projection system, the Stenographic 1970 system.

Luckily, today we have ROMPOS and TRANSDAT, which secure a millimeter precision, not a centimeter one but one that secures uniformity. It is not the purpose of this work to study the way in which the positions are obtained using the technology GNSS, so we will not go into further detail. We will remain at the level of studying the traversing method versus the method of the least squares.

In general, specialists in cadaster and topography are currently handling works at a smaller scale, mostly field tabulation documentations for small land owners. For the topographic upheavals they need a support network from which the detail points radiate.

The support network is translated into:

- traversing from the old points existing in the area;
- intersections from which a traversing starts towards the interest area;
- two points are being determined with the help of the GNSS technology from a traversing in the interest area is being conducted.

As it can be seen, the main method for transmitting the coordinates is traversing, either supported at both ends, closed on the point of departure, or in the wind. It is of course a method used to determine the coordinates, but which does not offer the solutions closest to the real value, since it is a theoretically requested.

COMPENSATING THE MEASURES THROUGH THE METHOD OF THE LEAST SQUARES

The method of the least squares is very well known, hence this is not the case to repeat all its formulas. Currently, due to the calculation system development, the Gauss method is no longer in use, instead, the conditional measurements and the matrix treatment are being used. Each measurement gives an equation, each new point gives two unknown (coordinates x and coordinate y). The number of measurement (equations) has to be larger than the number of unknown factors in order to obtain the most probable value of the coordinate as close as possible to the real value. Equations are of two or three kinds (on directions, on distances, and on GNSS determinations). In the study cases illustrated only two types of equations have been considered: based on directions, and based on distances. The general case used in processing the data is the method of the least squares, indirect measurements in which each measurement (on direction and on distance) gives out an equation. The equations are different for the two types of measurements, but have the unknown factor in common, respectively the coordinates x and y of the new points.

For both methods to obtain the position it is considered that the distance has been correctly reduced to the 1970 Stenographic projection plan.

Having the correction equations based on directions, distances and difference of coordinates, the following matrix relations can be written:

$$v = A x + l \quad (2.1)$$

$$N = A^T P A \quad (2.2)$$

$$x = -N^{-1} A^T P l \quad (2.3)$$

in which:

- v is the correction matrix;
- A is the matrix of the correction factor;
- x is the matrix of the unknown factor;
- P is the matrix of weighing factor.

The equations for distances have the following form:

$$\begin{aligned} v_{ij}^D &= A_{ij} dx_j + B_{ij} dy_j - A_{ij} dx_i - B_{ij} dy_i + l_{ij}^D \\ v_{ij}^D &= A_{ij} dx_j + B_{ij} dy_j - A_{ij} dx_i - B_{ij} dy_i + l_{ij}^D \end{aligned} \quad (2.4)$$

The free term is calculated as follows:

$$l_{ij}^D = D_{ij}^0 - D_{ij}^M = D_{ij}^0 - D_{ij}^M \quad (2.5)$$

And

$$A_{ij} = \frac{\Delta x_{ij}}{D_{ij}} = \cos \theta_{ij} A_{ij} = \frac{\Delta x_{ij}}{D_{ij}} = \cos \theta_{ij} \quad (2.6)$$

$$B_{ij} = \frac{\Delta y_{ij}}{D_{ij}} = \sin \theta_{ij} B_{ij} = \frac{\Delta y_{ij}}{D_{ij}} = \sin \theta_{ij} \quad (2.7)$$

The correction equations for directions have the following form:

$$\begin{aligned} v_{ij} &= -dZ_i + a_{ij}dx_j + b_{ij}dy_j - a_{ij}dx_i - b_{ij}dy_i + l_{ij} \\ v_{ij} &= -dZ_i + a_{ij}dx_j + b_{ij}dy_j - a_{ij}dx_i - b_{ij}dy_i + l_{ij} \end{aligned} \quad (2.8)$$

$$l_{ij} = \theta_{ij}^0 - \alpha_{ij}^0 - Z_{ij}^0 \quad l_{ij} = \theta_{ij}^0 - \alpha_{ij}^0 - Z_{ij}^0 \quad (2.9)$$

$$a_{ij} = -\rho^{cc} \frac{\sin \theta_{ij}^0}{D_{ij}^0} = -\rho^{cc} \frac{\Delta y_{ij}^0}{D_{ij}^0} a_{ij} = -\rho^{cc} \frac{\sin \theta_{ij}^0}{D_{ij}^0} = -\rho^{cc} \frac{\Delta y_{ij}^0}{D_{ij}^0} \quad (2.10)$$

$$b_{ij} = -\rho^{cc} \frac{\cos \theta_{ij}^0}{D_{ij}^0} = -\rho^{cc} \frac{\Delta x_{ij}^0}{D_{ij}^0} b_{ij} = -\rho^{cc} \frac{\cos \theta_{ij}^0}{D_{ij}^0} = -\rho^{cc} \frac{\Delta x_{ij}^0}{D_{ij}^0} \quad (2.11)$$

For each determined new point the precisions with which it has been calculated can be obtained. The forms are the following:

$$s_{x_i} = s_0 \sqrt{Q_{xixi}} s_{x_i} = s_0 \sqrt{Q_{xixi}} \quad , \quad \text{cu} \quad j=1,2 \quad \dots n \quad (2.12)$$

Quotients Q_{ij} are extracted from the main transverse of the matrix opposite to the normal system (matrix N^{-1})

DETERMINING THE COORDINATES BY USING THE TRASVERSING METHOD

The traversing method is a procedure largely used by the cadaster and geodesy specialists for areas with a small surface. Practically, it starts from a point with known coordinates, on top of which the total station is positioned, with another aimed point which also has known coordinates. From the stationary point, the next point of the traversing is aimed at, also a point that has known coordinates. The next point of the traversing is aimed at from the stationary point, correlated to the previously aimed point, thus the departure orientation can be determined. Also the forth and back distances can be measured (from the point with known coordinates to the new point, and then backwards).

The total station is then moved to the new point, from where the point of departure is being aimed, as well as to the next point of the traversing. The operation is being repeated until the traversing reaches the closing point, which also has the known position and regards the previous point of the traversing and another point with the known coordinates, for the closing on the orientation.

Practically, the coordinates are determined from one close up to another and are finally closed on an unknown point. The coordinates transmitted through the traversing should be close to the ones from the inventory, considering the precision of the old network and the errors accumulated in the traversing.

The general formulas of determination are:

$$\begin{aligned} x_B &= x_A + D_{0_{AB}} \cos \theta_{AB} \\ y_B &= y_A + D_{0_{AB}} \sin \theta_{AB} \end{aligned} \quad \begin{aligned} x_B &= x_A + D_{0_{AB}} \cos \theta_{AB} \\ y_B &= y_A + D_{0_{AB}} \sin \theta_{AB} \end{aligned} \quad (3.1)$$

The coordinates are being determined from one close up to another, finally reaching a point for known coordinates (the closing point). Between the sent coordinates and the coordinates already existing there is a difference that is called error of unclosing the traversing.

$$e_x = x_B^{Tr} - x_B e_x = x_B^{Tr} - x_B \quad (3.2)$$

$$e_y = y_B^{Tr} - y_B e_y = y_B^{Tr} - y_B$$

Total linear error:

$$e_t = \sqrt{e_x^2 + e_y^2} \quad (3.3)$$

This error must not overcome the imposed tolerance. The tolerance is given by the precision of the device with which is being worked. If the errors are within the tolerance limit, unitary corrections are being calculated:

$$C_x = -\frac{e_x}{D} C_x = -\frac{e_x}{D} \quad (3.4)$$

$$C_y = -\frac{e_y}{D} C_y = -\frac{e_y}{D}$$

In which D is the sum of the distances reduced to the 1970 Stenographic projection plan.

$$D = \sum D_0 D = \sum D_0 \quad (3.5)$$

The unitary correction is applied to each Δx and Δy according to distance. After applying the corrections, the coordinates are being retransmitted from one close up to another and

$x_B^{Tr} = x_B x_B^{Tr} = x_B$. Each point of the traversing will have new coordinates, corrected according to the traversing and the closing on the traversing.

CASE STUDY

A geodetic network has been created for the thickening whose beneficiary was the local Timis Waters Bazin Administration (Figure xx1). Within the network distances and directions have been determined. There have been 9 points with known coordinates and the new determined points have been 10. The number of equations was 89, enough to apply the method of the least squares and the indirect measures. The network has been compensated and the result have been the precisions presented in the table 4.1.

As the network has a linear configuration regarding the points that have been stationed with the total station, this has been considered as a traversing supported on both

ends (Figure xx2). We have calculated the coordinates with the known formulas and we have obtained the provisory positions. The traversing has been compensated and the unclosing has been -6,19 centimeters on the X axe, and 52,98 centimeters on the Y axe, respectively 53,34 centimeters total unclosing. The total length of the traversing has been of 5984,45 meters and the number of stations of 12. The starting point has been the point "La Gomila" and the closing have been on the point "Biled Nord". Both points are part of the national geodetic network .

Table 1

Compensated through the method of the least squares and the obtained precisions

Name of the point	X compensated	precision x	Y compensated	precision y	Total precision
	[m]	[mm]	[m]	[mm]	[mm]
BIS_SIRBEA	489381.005	0	193158.665	0	0
CA_SANDRA	499303.616	0	182951.287	0	0
BIS_BILED	495532.999	0	186890.594	0	0
BIS_SATCHINEZ	501354.193	0	192865.766	0	0
CAT_SATCHINEZ	500861.438	0	193459.266	0	0
LA_GOMILA	500021.708	0	189945.826	0	0
BIS_HODONI	496756.611	0	196534.092	0	0
BILED_NORD	496829.017	0	188278.994	0	0
BECIC	488937.319	0	193280.931	0	0
B33	499415.774	20	189689.407	23	30
B31	499875.812	20	190528.606	16	26
B30	500117.425	23	190729.652	18	29
B32	499445.179	22	190228.591	23	32
B34	499154.698	28	189222.829	30	41
B35	498864.445	32	188690.549	34	47
B36	498225.579	32	188442.562	37	49
B37	497616.381	27	188817.25	33	42
B38	497015.843	17	188654.847	15	22
B39	496808.699	18	188714.993	14	23
Medium error for determining a point: 25 mm					

After obtaining the coordinated in the two variants we have created the table (Table 4.2) with the values of the positions and the differences between the two variants.

Table 2.

Values and differences

Name point	compensated		traversing		dx[cm]	dy[cm]	d total [cm]
	x[m]	y[m]	x[m]	y[m]			
B39	496808.699	188714.993	496808.681	188715.035	1.8	-4.19	4.56
B38	497015.843	188654.847	497015.816	188654.906	2.72	-5.95	6.54
B37	497616.381	188817.25	497616.358	188817.239	2.33	1.127	2.59
B36	498225.579	188442.562	498225.488	188442.481	9.09	8.098	12.2
B35	498864.445	188690.549	498864.366	188690.427	7.91	12.2	14.5
B34	499154.698	189222.829	499154.65	189222.707	4.85	12.21	13.1

B33	499415.774	189689.407	499415.748	189689.287	2.64	11.97	12.3
B32	499445.179	190228.591	499445.162	190228.505	1.75	8.596	8.77
B31	499875.812	190528.606	499875.796	190528.539	1.57	6.723	6.9
B30	500117.425	190729.652	500117.416	190729.603	0.92	4.927	5.01

CONCLUSIONS

The case presented is an example proving there are large differences influencing the coordinates of the radiated points between the coordinated determined by the traversing and those determined by the method of the least squares, the indirect measurements.

The radiated points often represent coins of property defining edges. If two properties are being measured by two different specialists, and they will obtain those coordinates in a different way using another method of calculation, subsequently also the coordinates of the common points will be substantially different. We are also not mentioning the different techniques used to measure the land, which can also lead to errors of positioning.

In general, authorized cadaster specialists use the method of traversing because it is easier to calculate. This is wrong since currently, with the help of evolved computers, we can work the data very easy. Let us take for instance the Excel program, delivered along with the computer. This program allows us to work with needed matrixes regarding the least squares method. We have to know that a not very large network is being developed usually, this only for a certain property. Further more, the calculation will be very simple.

From the table 4.2, we can see that the differences of coordinates are smaller at the heads and bigger in the middle, as it was normal as the constriction is done at the heads.

It can be noticed that the total error in the middle is over 13 centimeters.

The conclusion is that it will be necessary to obtain the final coordinates using the methods of the least squares, which offers a value much closer to the real one.

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