

SISTEMS OF ALTITUDES USED IN GEODESY

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ABSTRACT

In order to understand the consequences of the parallel misalignment of the level surfaces for the geodetic work, we should take for instance the system of orthometric altitudes, in which the geoid is the reference surface and the orthometric altitudes is the segment of force line caught between the position of the point, on the earth surface and respectively on the geoid.

1. CONSEQUENCES OF THE PARALLEL MISALIGNMENT OF THE EQUIPOTENTIAL SURFACE

In the ellipsoid geodesy, a system of altitudes is defined as follows:

- by choosing a surface of reference;
- by adopting a definition with a physical or geometric meaning, through which the position of the points on the surface of the Earth is described against the surface of reference;
- by transforming the bench-marks, measured through the geometrical levelling, in the considered system of altitudes.

As known, the levelling surfaces are not parallel, hence in each point of space can be written:

$$dW = -gdh$$

a formula that established the dependency between the distance dh and the potential difference dW existent between two infinitely close levels.

In order to understand the consequences of the parallel misalignment of the level surfaces for the geodetic work, we should take for instance the system of orthometric altitudes, in which the geoid is the reference surface and the orthometric altitudes is the segment of force line caught between the position of the point, on the earth surface and respectively on the geoid.

On figure 1, we can see that the sum of the elementary levelling differences, measured on the lay-out between points A and B, noted $\sum_A^B \Delta h = \Delta h_{AB}$ is not equal to the difference

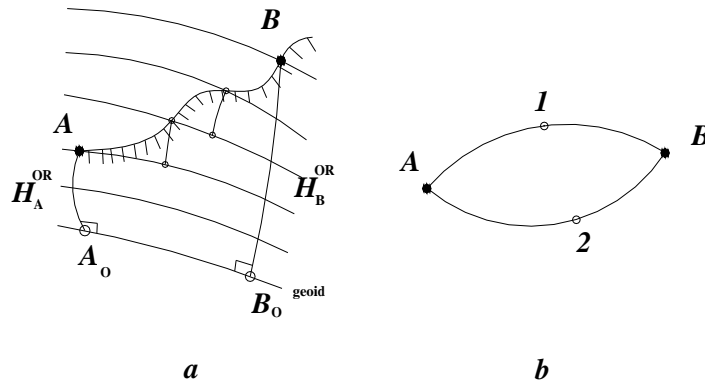


Fig. 1. The consequences of the misalignment of the level surfaces on the level callibrations for lines and large syze polygons.

The orthometric altitudes of points A and B, are noted H_A^{or} and H_B^{or} .

Thus, it is shown how the result obtained directly throught the works of geometric levelling $\sum_A^B \Delta h$ is in relation to the covered layout. The conclusion is in fig. 1 the sum of the elementary level differences, measured on layouts 1 and 2, which will not be equal to each other, not even in the ideal case of the perdedted geodesic observations with no errors of measurement. In the formed polygon there will be a resulted unclosing, also called *fundamental error of the geometric geodetic levelling*.

This is why in the works of geometric levell of superior order it is necessary to also conduct gravimetric analysis parallely in order to calculate the differences of potential.

2. THE SYSTEM OF DYNAMIC ALTITUDES

The notion of dynamic altitude has been introduces by Helmert in the year 1873. If we admit that we have a level line drawing from the fundamental zero point, to point P, where the level differences as well as the gravity acceleration has been measured, the difference C_P between the potential of the geoid W_0 , and the potential of the level surface W_P of the point P is called the geopotential number of point P.

The geopotential number can be determined throught conducting the two types of works:

- gravimetric works (g);
- geometrical levelling works (h).

The dynamic altitude is noted with H^d and is obtained through the division of the geopotential number C_P by a constant value, more precisely by the value of the normal gravity, at the 45° latitude, in relation to the reference elipsoid:

$$H_P^D = \frac{C_P}{\gamma_{45^\circ}}$$

The system of dynamic altitudes is characterised by a special property, i.e.: the points situated on a certain level surface have the same dynamic altitude.

The reference surface is the geoid with the known potential W_0 , and the dynamic altitude of the point P (H_P^D) is defined by the above relation, where γ_{45° is the normal gravity at the level of the elipsoid at latitude $B=45^\circ$; $\gamma_{45^\circ}=980617.6$ mgali.

3 DYNAMIC CORRECTION

The dynamic correction on the AB layout is noted with u_{AB}^d and its size is calculated by the formula:

$$u_{AB}^D = \int_A^B \frac{g - \chi_{45^\circ}}{\chi_{45^\circ}} dh \approx \sum_A^B \frac{G - \chi_{45^\circ}}{\chi_{45^\circ}} \Delta h$$

The system of dynamic coordinates has been at the based of the creation of the level network in Western Europe.

4 The system of orthometric altitudes

Since the definition of the geopotential number does not depend on the used layout, it is supposed that the integration is conducted along the force line:

$$C_P = \int_O^{H_P^{OR}} g dH,$$

the definition of the orthometric altitude is: where \bar{g} represents the average of the gravity values along the force line $P_O P$ when a finite number of segments is considered.

$$H_P^{or} = \frac{1}{\bar{g}} C_P$$

5 The orthometric correction

The orthometric correction is added to the level difference measured directly in order to determine the orthometric level difference by the formula:

$$\Delta H_{AB}^{OR} = H_{AB}^D + (H_B^{OR} - H_A^D) - (H_B^{OR} - H_A^{DR})$$

In order to calculate the difference existent between the orthometric and the dynamic altitudes we imagine a geometric leveling layout going from point A_0 , along the force line, up to mark A. It can be described as: $\sum_{A_0}^A \Delta h = H_A^{OR}$

On the same layout $\Delta H_{A_0}^D = H_A^D = \sum_{A_0}^A \Delta h + u_{A_0A}^D$

$$H_A^D = H_A^{OR} + u_{A_0A}^D \quad \text{or} \quad H_A^{OR} - H_A^D = -u_{A_0A}^D \quad \text{and similarly} \quad H_B^{OR} = H_B^D = -u_{B_0B}^D$$

The dynamic corrections are calculated by the known formulars.

The level differences from A to B are calculated by the formula $\Delta H_{AB}^{ORr} = \Delta h_{ab} + u_{ab}^{or}$ where u_{AB}^{oOR} represents the orthometric correction on the layout AB.

6 Helmert Altitude

$$H_P^H \approx \frac{C_P}{g + 0.0424H} \quad \text{where the numerator represents the}$$

geopotential number and the denominator, the medium gravity.

7 The ellipsoid orthometric altitude

Has been very used in the beginning of geodesy when gravimetric measurements were not available. If in the formula of the orthometric correction, we substitute $g \equiv \chi$, we obtain the expression of the ellipsoid orthometric correction:

$$u_{ab}^{ORS} = \sum_A^B \frac{\chi - \chi_{45^\circ}}{\chi_{45^\circ}} \Delta h + \frac{\bar{\chi}_A - \chi_{45^\circ}}{\chi_{45^\circ}} H_A - \frac{\bar{\chi}_B - \chi_{45^\circ}}{\chi_{45^\circ}} H_B$$

8 The system of normal altitudes

The system of normal altitudes is currently being used in our country and it has been grounded by M. S. Molodenski in the year 1945.

Considering that by knowing the medium gravity \bar{g} along the force line, Molodenski suggest to use the normal gravity field instead of the real gravity field.

The formula to calculate the normal altitude of a point P will be:

$$H_P^N = \frac{1}{\bar{\chi}} C_P \quad \text{where} \quad \bar{\chi} = \frac{1}{H^n} \int_0^{H^n} \chi dH^n$$

The level difference in the system of normal altitudes will be:

$$\Delta H_{AB}^N = H_B^N - H_A^N = \Delta h_{AB} + u_{AB}^N$$

Where the correction in the system of normal altitudes is:

$$u_{AB}^n = \sum_A^B \frac{g - \chi}{\chi_{45^\circ}} \Delta h + u_{AB}^{ors}$$

CASE STUDY:

Determining the differences of altitudes in different systems.

Initial data:

- The latitude of bench 1 : $B_1=45^\circ 23' 19.444''$
- The latitude of geodesic bench 2 : $B_2=45^\circ 24' 16.6667''$
- The approximate altitude of bench 1 : $H_1^* = 325.85$ m
- The gravity of bench 1 : $g_1 = 980615.62$ mgal.
- The gravity of bench 2 : $g_2 = 980654.60$ mgal.
- The measured difference of level: $h_{12}^0 = 16.7634$ and $h_{12}^{45^\circ} = 980617.6$ mgal.

It is requested:

- The Difference of altitudes in the systems:
 - dynamic;
 - normal orthometric (spheroid orthometric);
 - normal (Molodenski).

Answer:

-The Normal Gravity on the ellipsoid of the benchmarks 1 and 2

$$g_1^{1980} = 978032.7 (1 + 0.00527904 \sin^2 B_1) = 980635.14 \text{ mgal.}$$

$$g_2^{1980} = 978032.7 (1 + 0.00527904 \sin^2 B_2) = 980636.01 \text{ mgal.}$$

-The normal gravity at ground level

$$g_1^H = g_1^{1980} - 0.086 H_A^* = 980534.48 \text{ mgal.}$$

$$g_2^H = g_2^{1980} - 0.086 H_A^* = 980530.28 \text{ mgal.}$$

-The difference of altitude in the dynamic system

$$u_{12}^{ors} = \frac{1}{\chi_{45^\circ}} \left[\frac{g_1 + g_2}{2} - \chi_{45^\circ} \right] \Delta h_{12}^o = 0.0015 [m]$$

$$\Delta H_{12}^d = \Delta h_{12}^o + u_{12}^d = 16.764 [m]$$

-The difference of altitude in the normal orthometric system

$$u_{12}^{ors} = \frac{1}{\chi_{45^\circ}} \left[\left(\frac{\chi_1 + \chi_2}{2} - \chi_{45^\circ} \right) \Delta h_{12}^o + \left(\frac{\chi_1 + \chi_1^H}{2} - \chi_{45^\circ} \right) H_1^* - \left(\frac{\chi_2 + \chi_2^H}{2} - \chi_{45^\circ} \right) H_2^* \right] = 0.0003 [m]$$

$$\Delta H_{12}^{ors} = \Delta h_{12}^o + u_{12}^{ors} = 16.7631 [m]$$

-The difference of altitude in the normal system

$$u_{12}^n \cong u_{12}^d + u_{12}^{ors} = 0.0012 [m]$$

$$\Delta H_{12}^n = u_{12}^n + \Delta h_{12}^o = 16.7646 [m]$$

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