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# ANALYSIS OF THE STATE OF STRESSES AND STRAINS IN THE CURVED STILT, USING TRIANGULAR FINITE ELEMENTS (CST)

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## ABSTRACT

*The paper presents a study on the analysis of the distribution of stresses and strains in the curved stilt of the plough-body, using the Finite Element Method. Modeling was performed with the help of triangular finite elements "CST" (Constant Strain Triangle), using a special program for this type of applications "CSTPL.EXE", developed at University Politehnica of Bucharest, Department of Materials Strength.*

## INTRODUCTION

The triangular finite elements with three "CST" nodes are among the simplest, using linear interpolation functions for modeling, and being used successfully in the analysis of plane structures (2D).

Analiza din prezenta lucrare scoate în evidenţă faptul că distribuţia tensiunilor în bârsa curbă impune utilizarea unor secţiuni a căror grosime în zona fibrei medii deformate trebuie să fie mai mică decât grosimea din zona conturului bârsei unde tensiunile sunt mult mai mari (Biriș S. Șt. 1999; Biriș S. Șt., 2007).

## MATERIAL AND METHOD

General considerations for triangular finite elements (CST)

One of the simplest elements for the analysis of two-dimensional structures, is the triangular element that uses linear interpolation functions for modeling (Figure 1).

Using the CST (Constant Strain Triangle) element, study plane states of stress can be studied (for example, in the case of thin plates with loads in the x-y plane and not perpendicular to it) or plane states of strains (in the case of structures with geometry and constant load on largest size direction) (Blumenfeld et al, 1992; Căproiu et al, 1982; Blumenfeld M., 1995).

Displacements are modelled using a linear shape function:

$$\Phi(x,y)=\alpha_1+\alpha_2{}^3x+\alpha_3{}^3y \quad (1)$$

or written using the following form of functions:

$$\Phi(x,y)=N_i(x,y)\cdot\Phi_i+N_j(x,y)\cdot\Phi_j+N_k(x,y)\cdot\Phi_k \quad (2)$$

where:

$$N_i(x,y)=\frac{1}{2\cdot A^{(e)}}\cdot\left(a_i+b_i\cdot x+c_i\cdot y\right) \quad (3)$$

and:

$$\begin{aligned} a_i &= x_j\cdot y_k - x_k\cdot y_j \\ b_i &= y_j - y_k \\ c_i &= x_k - x_j \end{aligned} \quad (4)$$

The area of the element was denoted by $A^{(e)}$:

$$A^{(e)}=\frac{1}{2}\cdot\begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} \quad (5)$$

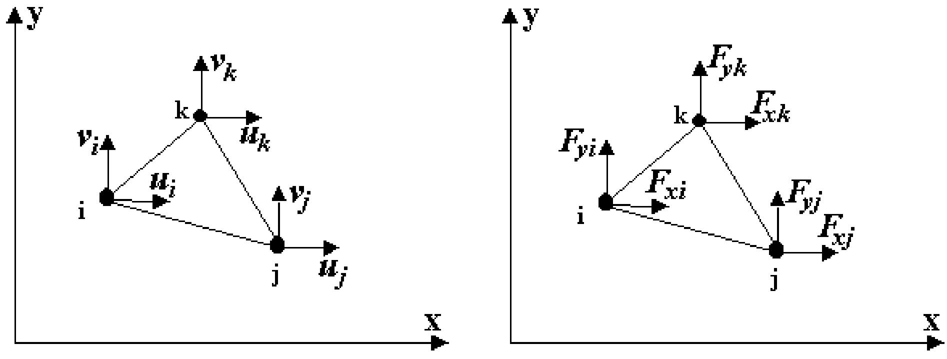



***Fig. 1. The CST element***

The connection between strains and displacements is given by the matrix î$B^{(e)}$ş:

$$\left[B^{(e)}\right]=\frac{1}{2\cdot A^{(e)}}\cdot\begin{bmatrix} b_i & 0 & b_j & 0 & b_k & 0 \\ 0 & c_i & 0 & c_j & 0 & c_k \\ c_i & b_i & c_j & b_j & c_k & b_k \end{bmatrix} \tag{6}$$

The connection between stresses and strains (for an isotropic material), in the case of the plane state of stresses, can be written as follows:

$$[D]=\frac{E}{1-\upsilon^2}\cdot\begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon}{2} \end{bmatrix} \tag{7}$$

respectively, in the case of the plane state of strains:

$$[D]=\frac{E}{(1-\upsilon)\cdot(1-2\cdot\upsilon)}\cdot\begin{bmatrix} 1-\upsilon & \upsilon & 0 \\ \upsilon & 1-\upsilon & 0 \\ 0 & 0 & \frac{1-2\cdot\upsilon}{2} \end{bmatrix} \tag{8}$$

where: E - the longitudinal modulus of elasticity;
$\nu$ - coefficient of transverse contraction.

Since the elements of the matrix (6) are constant, the stiffness matrix of the element is finally obtained, in the following form:

$$\left[K^{(e)}\right]=\left[B^{(e)}\right]^T\cdot[D]\cdot\left[B^{(e)}\right]\cdot A^{(e)}\cdot t^{(e)} \tag{9}$$

where $t^{(e)}$ is the thickness of the element, and thus:

$$\left[K^{(e)}\right]\cdot\left\{\delta^{(e)}\right\}=\left\{F^{(e)}\right\} \tag{10}$$

where:

$$\left\{\delta^{(e)}\right\}^T=\left\{u_i \quad v_i \quad u_j \quad v_j \quad u_k \quad v_k\right\} \tag{11}$$

is the vector of nodal displacements, and:

$$\left\{F^{(e)}\right\}^T=\left\{F_{xi} \quad F_{yi} \quad F_{xj} \quad F_{yj} \quad F_{xk} \quad F_{yk}\right\} \tag{12}$$

represents the vector of nodal forces.

The main features of the CST element are:

- is generated by three nodes i, j and k (Figure 1);
- has two degrees of freedom per node (translation along the x and y axis);
- has constant thickness;
- can be loaded with forces at the nodes (Zinkiewich O.C, 1977; Cârdei et al, 2012; Sorohan Şt., 1996).

## RESULTS AND DISCUSSIONS

The curved stilt of the plough-body, shown in Figure 2, is loaded in the working process by a maximum resistance force from the soil F=10 kN. Knowing that the curved stilt is made of steel, for which the longitudinal modulus of elasticity is $E=2{,}1\cdot10^5$ MPa, the Poisson's ratio $\nu$=0.3 and the thickness t=50 mm, the aim is to study the stress state and strains distribution (Zhang Z., 2020; Ungureanu et al, 2016).

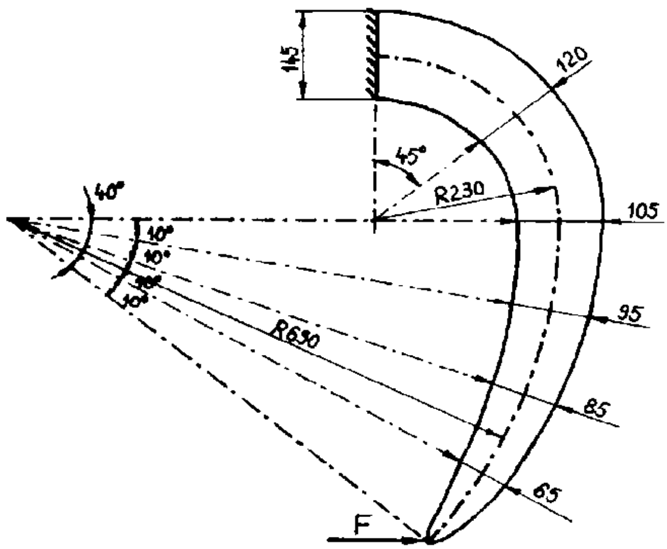
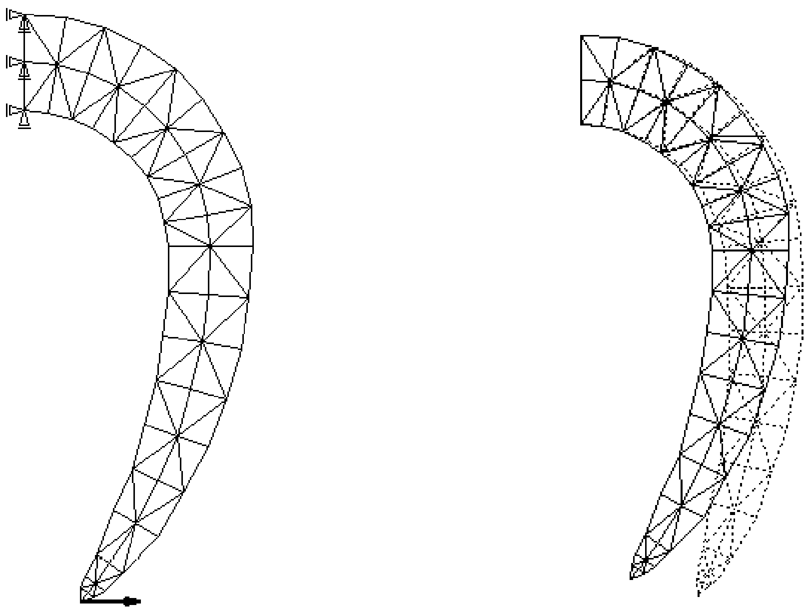



***Fig. 2. The curved stilt of the plough-body***

The discretization with triangular finite elements of the curved stilt structure is presented in Figure 3 a), and contains a number of 58 nodes, respectively 74 elements.

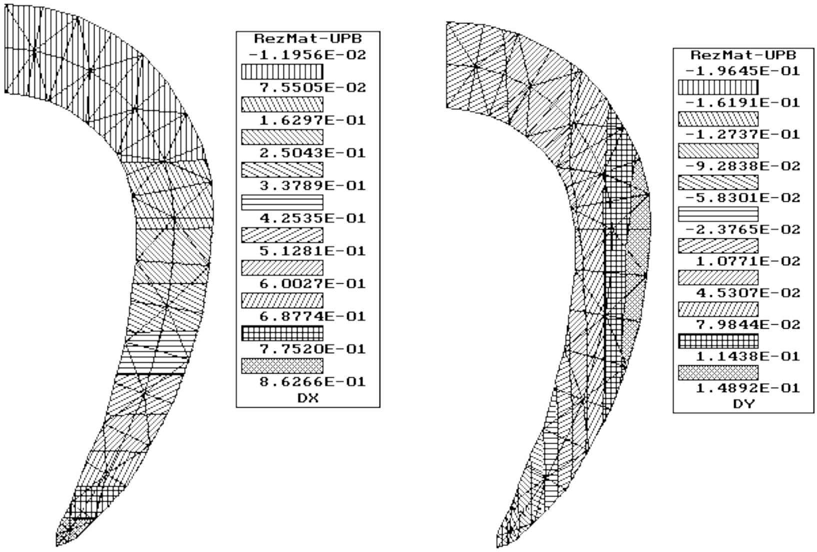


a) b)

***Fig. 3 a). Discretization of curved stilt, b) Deformed***

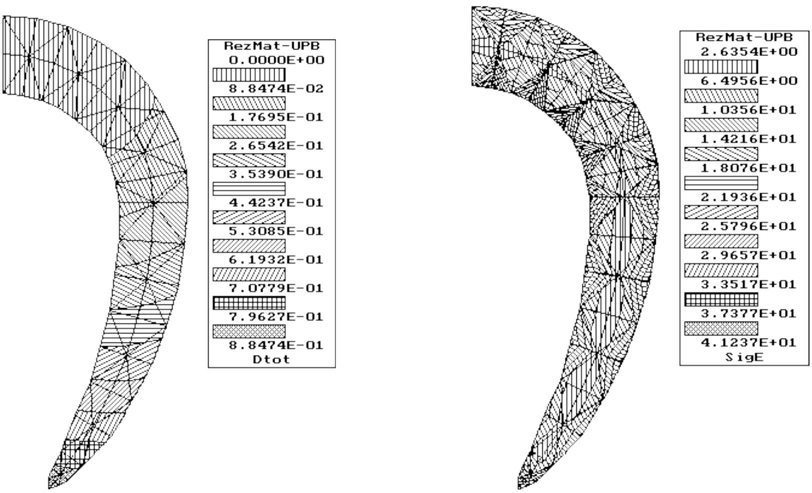
By running the CSTPL.EXE program, realized in the University Politehnica of Bucharest, Department of Materials Strength, the nodal displacements and the distribution of stresses from the curved stilt are obtained.

The deformed curved stilt is shown in Figure 3 b). The results obtained by solving the problem are presented graphically. Thus, in Figure 4 a) are presented the nodal displacements on the X direction, in Figure 4 b) the nodal displacements along the Y direction, and in Figure 5 a) the total nodal displacements, (Vlăduţ V, 2012). Figure 6 b) shows the distribution of Sech equivalent stresses ( $S_{ech} = \sqrt{S_1^2 + S_2^2 - S_1 \cdot S_2}$ ), in Figure 6 a) are presented the equine stress lines for the equivalent stresses, and in Figure 6 b) the orientation of the main stresses, in scale representation, for the curved stilt. The graph in Figure 7 shows the variation of equivalent stresses in the embeddedment (Bao et al, 2019, Jianfei et al, 2021).




**a) b)**

***Fig. 4. a) Nodal displacements DX , b). Nodal displacements DY***




**a) b)**

***Fig. 5. a) Total nodal displacements, b) Distribution of equivalent stresses***

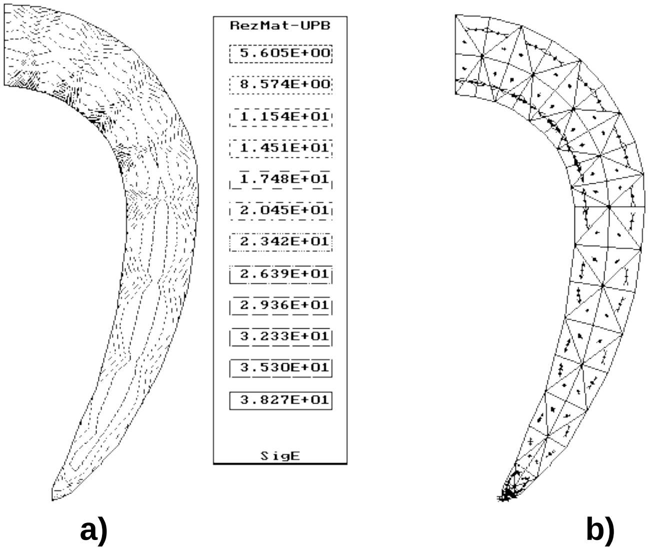



***Fig. 6. a) Equine stress lines for equivalent stresses, b) Orientation of main stresses***

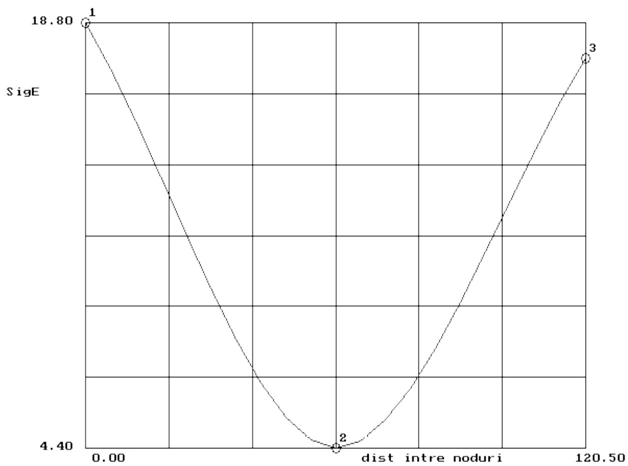



***Fig. 7. Variation of equivalent stresses in the embedded nodes***

## CONCLUSIONS

1. The analysis of the state of stress and strains in the curved stilt of the plough-body can be performed using triangular finite elements, with three nodes (CST).
2. Due to the interaction with the soil, it can be observed that the tip of the curved stilt supports the largest displacements (Figure 5 a).

3. The model developed in this paper allows the analysis of the stress distribution from any point on the curved stilt (Figure 5 b).
4. The highest stresses are found in the area of the contour of the curved stilt and the lowest in the area of the deformed medium fiber, which determines, for optimization, the decrease of stilt thickness in the middle area, respectively the increase of stilt thickness at the peripheries.

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